REPRESENTATION OF THE PROCESSES INVOLVED IN THE TEMPERATURE FLUCTUATIONS EXPERIENCED BY THE WALLS OF BUILDINGS ON THE BASIS OF THE HEAT ASSIMILATION COEFFICIENT $(a/\omega)^{1/2}$

V. V. Nasedkin

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The quantities characterizing the temperature fluctuations taking place in multilayered walls are expressed in the form of the so-called "heat-assimilation coefficient." An approximate method of determining the damping of the temperature fluctuations is proposed.

In solving problems of the kind envisaged by operational methods, the amplitude of the temperature fluctuations in the cross section of the wall may be expressed [1] in the form

$$t_m(x) = t_m (N_i N_{-i})^{1/2}, \tag{1}$$

where N_i and N_{-i} are determined by the conditions of the problem. For a single-layered wall these are equal to [2]

$$N_{\pm i} = \frac{\frac{\lambda}{\alpha_{in}}\sqrt{\pm i\frac{\omega}{a}}\,\mathrm{ch}\,x\sqrt{\pm i\frac{\omega}{a}} + \mathrm{sh}\,x\sqrt{\pm i\frac{\omega}{a}}}{\left(\frac{\lambda}{\alpha_{in}}\sqrt{\pm i\frac{\omega}{a}} + \frac{\lambda}{\alpha_{in}}\sqrt{\pm i\frac{\omega}{a}}\right)\mathrm{ch}\,\delta\sqrt{\pm i\frac{\omega}{a}} + \left[1 + \frac{\lambda^2}{\alpha_{in}\alpha_{out}}\left(\sqrt{\pm i\frac{\omega}{a}}\right)^2\right]\mathrm{sh}\,\delta\sqrt{\pm i\frac{\omega}{a}}}.$$

We see from Eq. (2) that the same quantity $(a/\omega)^{1/2}$ enters into the arguments of the hyperbolic functions and into the coefficients of these functions. Lykov defined this quantity as the "heat-assimilation coefficient" [1]

$$\sqrt{\frac{a}{\omega}} = \xi. \tag{3}$$

According to Eq. (3) the quantity ξ is characterized by the thermophysical properties of the material and the cyclic frequency of the thermal flux fluctuation. By its very nature this quantity constitutes a parameter of the heat-transfer process associated with harmonic temperature variations. The quantity ξ has the dimensions of length (m).

We may express the complexes of quantities entering into Eq. (2) in the following way:

$$x\sqrt{\frac{\omega}{a}} = \frac{x}{\xi} = D_x, \tag{4}$$

$$\delta\sqrt{\frac{\omega}{a}} = \frac{\delta}{\xi} = D, \tag{5}$$

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$$\lambda \sqrt{\frac{\omega}{a}} = \frac{\lambda}{\xi} = S, \tag{6}$$

where D_X is the relative coordinate, and D is the relative thickness of the layer.

The dimensionless quantities D_X and D are generalized variables. Gukhman showed [3] that any generalized variable might be represented in the form of a ratio between two similar quantities. The denominator of such a ratio has become known as the "characteristic scale."

In Eqs. (4) and (5) the characteristic scale is ξ.

The quantity S is well known in building thermophysics as the ratio of the thermal-flux amplitude to the temperature amplitude at the surface of a thick layer:

$$\frac{q_m(x)}{t_m(x)} = \sqrt{\lambda c \gamma \omega} = S. \tag{7}$$

This interpretation of S was first given by Vlasov [4]. Following the investigations of Vlasov and Muromov [5], Shklover developed a method of approximately determining the damping of the temperature fluctuations in the walls of buildings [6]. In Shklover's equations the damping of the temperature fluctuations is expressed in the form of a function of the quantities S, Y, and D = SR. Here Y (like S) is the ratio of thermal flux and temperature amplitudes, but measured at the surface of a thin layer, at which the laws of a semiinfinite solid are broken and this ratio ceases to be a constant quantity.

Using the parameter ξ as characteristic scale, the physical nature of S and Y may be expressed in another way.

According to (6), S is the thermal conductivity of a layer of thickness ξ. For $\Delta x = \xi$ the relative thickness of the layer equals the dimensionless unit ($\Delta D_X = 1$).

Thus S is the thermal conductivity of a layer having a relative thickness of unity.

We express (5) in the form

$$D = \delta \sqrt{\frac{\omega}{a}} = \frac{\delta}{\lambda} \lambda \sqrt{\frac{\omega}{a}} = SR = \frac{R}{\frac{1}{S}}. \tag{8}$$

If we understand S as the thermal conductivity of a unit dimensionless layer, then according to (8) the dimensionless thickness D is the ratio of the thermal resistance of a particular layer to the resistance of a layer of unit thickness.

Equation (8) may be obtained directly from the equation $\delta = \lambda R$ on rewriting the latter after allowing for the use of relative coordinates

$$\frac{\delta}{\xi} = \frac{\lambda}{\xi} R, \tag{9}$$

and then

$$D = SR. \tag{8a}$$

The thermal flux and the boundary condition of the fourth kind are here expressed by the equations

$$q = -\lambda \frac{\partial t}{\partial x} = -\frac{\lambda}{\xi} \left(\frac{\partial t}{\partial \frac{x}{\xi}} \right) = -S \frac{\partial t}{\partial D_x} = -S \,\mathrm{tg}\, \psi, \tag{10}$$

$$\frac{\mathrm{tg}\, \psi_{n+1}}{\mathrm{tg}\, \psi_n} = \frac{S_n}{S_{n+1}}. \tag{11}$$

It follows from Eqs. (8a) and (10) that, on passing to relative coordinates, the quantity S, being a proportionality factor, plays the part of the thermal conductivity λ. On this basis S may be defined as the cyclical thermal conductivity ($W/m^2 \cdot K$).

We see from (11) that, if we take the parameter ξ as the unit measured along the horizontal axis, the ratio between the slopes of the tangents to the temperature curve at the interface between the layers is inversely proportional to the ratio between the cyclical thermal conductivities.

In order to derive approximate formulas for the damping of the temperature fluctuations in the layer, we shall assume that Eq. (11) also extends to the temperature amplitude.

From Eq. (11) we find

$$\operatorname{tg}\psi_n = \frac{\frac{1}{S_n}}{1}, \tag{12}$$

$$\operatorname{tg}\psi_{n+1} = \frac{\frac{1}{S_{n+1}}}{1}. \tag{13}$$

According to (12) and (13) the angles ψ_n and ψ_{n+1} may be obtained if we set off segments equal to the dimensionless unit along the horizontal axis and distances reciprocal to the cyclical thermal conductivities S_n and S_{n+1} along the vertical axis (Fig. 1a) [2]. If the layer n is "thin" ($D_n < 1$), then not only the layer n, but also (to some extent) the layer $n-1$, lies within the thickness of the unit section AB (Fig. 1b). The thermal conductivity of such an inhomogeneous unit section we may call Y_n. Let us express this as the sum of the thermal conductivities of the sections AL and LB:

$$Y_n = Y_{n-1}(1 - D_n) + S_n D_n.$$

Analysis shows that Eq. (14) is only valid for $S_n/Y_{n-1} \geq 1$. If $S_n/Y_{n-1} < 1$, then

$$\frac{1}{Y_n} = (1 - D_n)\frac{1}{Y_{n-1}} + D_n\frac{1}{S_n}. \tag{15}$$

Since $(1-D_n)(1/Y_{n-1}) = R_{AL}$ and $D_n(1/S_n) = R_n = R_{LB}$, Eq. (15) reduces to the form

$$(R_{AL} + R_{LB})Y_n = 1. \tag{16}$$

This latter form has the sense of Eq. (8a) in which D = 1. For $D_n \geq 1, Y_n = S_n$ [6].

Thus, if of two continuous layers n and n + 1 layer n is thin ($D_n < 1$), Eq. (11) will take the following form for purposes of approximate calculations

$$\frac{\operatorname{tg}\psi_{n+1}}{\operatorname{tg}\psi_n} = \frac{Y_n}{S_{n+1}}. \tag{17}$$

According to [2] the damping of the temperature fluctuations over the thickness of the layer may be approximately represented in the following way: At the inner surface of the layer, within a region of relative thickness unity, the change in amplitude takes place linearly; subsequently it proceeds exponentially right up to the outer surface, as in a semiinfinite solid (Fig. 2a). If the relative thickness of the layer is less than unity, the change of amplitude is approximated by a straight line throughout the whole thickness.

Let us consider the damping of the temperature fluctuations in a "thin" layer. In accordance with Eq. (12) we set off a segment AB equal to unity, proceeding from the inner surface of the layer along the horizontal axis (Fig. 2b). The damping of the temperature fluctuations in layer n is expressed by the equation

$$\nu_n = \frac{NM + MC}{BP} = \frac{\operatorname{tg}\psi_n}{\operatorname{tg}\psi_{n-1}} D_n + 1. \tag{18}$$

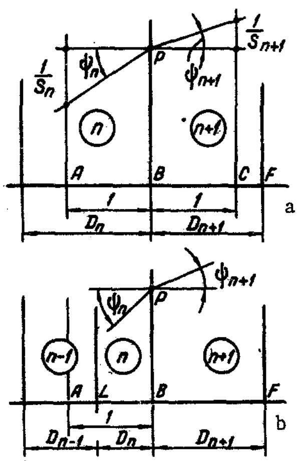

Fig. 1

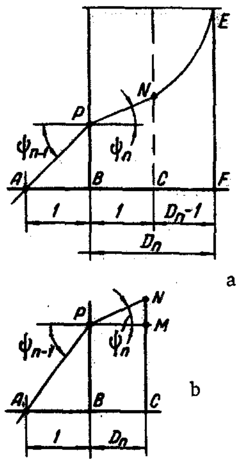

Fig. 2

Fig. 1. Diagram illustrating the meaning of Eqs. (11), (12), (13) (a) and Eq. 17 (b).

Fig. 2. Damping of temperature fluctuations for $D_n > 1$ (a) and $D_n \leq 1$ (b).

Allowing for (17), we finally obtain

$$\nu_n = \frac{Y_{n-1}D_n + S_n}{S_n}. \tag{19}$$

In a layer of considerable thickness ($D_n > 1$; Fig. 2a) the damping of the temperature fluctuations is expressed by the equation

$$\nu_n = \nu_{BC}\nu_{CF}. \tag{20}$$

We find the damping of the fluctuations in unit section BC from Eq. (19):

$$\nu_{BC} = \frac{Y_{n-1} + S_n}{S_n}. \tag{21}$$

We determine the damping of the fluctuations beyond the unit section as in a semiinfinite solid:

$$\nu_{CF} = \exp\left(\frac{D_n - 1}{\sqrt{2}}\right) = 0.5\exp\left(\frac{D_n}{\sqrt{2}}\right). \tag{22}$$

As a result of this, the damping of the temperature fluctuations in layer n of a multilayered wall will be given by the following equations:

for $D_n > 1$

$$\nu_n = 0.5\exp\left(\frac{D_n}{\sqrt{2}}\right)\left[\frac{Y_{n-1} + S_{\text{out}}}{S_n}\right], \tag{23}$$

for $D_n \leq 1$ – by Eq. (19).

For the first layer, according to [6, 7] $Y_{n-1} = Y_0 = \alpha_B$, and hence for $D_1 > 1$

$$\nu_1 = 0.5\exp\left(\frac{D_1}{\sqrt{2}}\right)\left[\frac{\alpha + S_1}{S_1}\right], \tag{24}$$

for $D_1 \leq 1$

$$\nu_1 = \frac{\alpha D_1 + S_1}{S_1}. \tag{25}$$

The damping of the temperature fluctuations on passing from the outer air to the inner surface of the wall is obtained from Eq. (19) if we take $S_n = \alpha_{out}$ and $D_n = D_{out} = 1$ [7]:

$$\nu_{out} = \frac{\alpha_{out} + Y}{\alpha_{out}}. \quad (26)$$

The quantity Y_n in (19) and (23)-(26) is the thermal conductivity of an inhomogeneous unit layer. This quantity is determined by the expressions:

for layer n

$$Y_n^{\pm 1} = Y_{n-1}^{\pm 1}(1 - D_n) + S_n^{\pm 1} D_n, \quad (27)$$

for the first layer

$$Y_1^{\pm 1} = \alpha_в^{\pm 1}(1 - D_1) + S_1^{\pm 1} D_1. \quad (28)$$

In (27) and (28) the power indices are positive for $S_n/Y_{n-1} \geq 1$, $S_1/\alpha_{in} \geq 1$ and negative if these ratios are smaller than unity. For $D_n \geq 1$

$$Y_n = S_n. \quad (29)$$

Equations (23), (24), (26), and (29) are the same as in the method of Shklover. The error of the approximation in the method described in this paper is also the same as that of the Shklover calculation (± 15%).

NOTATION

a, thermal diffusivity; λ, thermal conductivity; c, specific heat; γ, bulk mass of the material; ω, cyclical frequency of the fluctuations; x, coordinate; δ, thickness of layer; R, thermal resistance; t_m, amplitude of the temperature fluctuation in the outer air; $t_m(x)$, $q_m(x)$, amplitudes of the temperature and thermal flux fluctuations in section x; α_{out}, α_{in}, heat-transfer coefficients of the outer and inner surfaces of the wall.

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